Optimal Synthesis of Array Pattern for Concentric Circular Antenna Array Using Hybrid Evolutionary Programming

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***Abstract*—In this paper one optimization heuristic search technique, Hybrid Evolutionary Programming (HEP) is applied to the process of synthesizing three-ring Concentric Circular Antenna Array (CCAA) focused on maximum sidelobe-level reduction. This paper assumes non-uniform excitations and uniform spacing of excitation elements in each three-ring CCAA design. Experimental results reveal that the design of non-uniformly excited CCAAs with optimal current excitations using the method of HEP provides a considerable sidelobe level reduction with respect to the uniform current excitation with d=λ/2 element-to-element spacing. Among the various CCAA designs, the design containing central element and 4, 6 and 8 elements in three successive concentric rings proves to be such global optimal design with global minimum SLL (-40.22 dB) as determined by HEP.**

***Index Terms*—Concentric Circular Antenna Array, Non-uniform Excitation, Sidelobe Level, Hybrid Evolutionary Programming**

I. INTRODUCTION

In array pattern synthesis, the main objective is to find the physical layout of the array that produces the radiation pattern closest to the desired pattern. Over the past few decades [1-8] many synthesis methods are concerned with suppressing the SLL while preserving the gain of the main beam. Low sidelobes in the array factor are usually obtained through amplitude excitation weighting the signal at each element. The antenna arrays have been widely used in phase array radar, satellite communications and other domains [6]. In the satellite communications in order to improve the ability of antenna array to resist interference and noise the pattern of the antenna array should have low sidelobes, controllable beamwidth and the pattern synthesis in azimuth angles. The traditional optimization methods cannot bear the demand of such complex optimization problem.

Classical optimization methods have several disadvantages such as: i) highly sensitive to starting points when the number of solution variables and hence the size of the solution space increase, ii) frequent convergence to local optimum solution or divergence or revisiting the same suboptimal solution, iii) requirement of continuous and differentiable objective cost function, iv) requirement of the piecewise linear cost approximation, and v) problem of convergence and algorithm complexity. So, in this work, for the optimization of complex, highly non-linear, discontinuous, and non-differentiable array factors of CCAA designs, one heuristic search technique such as a novel hybrid evolutionary programming technique (HEP) [9] is adopted. Each optimal CCAA design should have an optimized set of non-uniform current excitation weights, which, when incorporated, results in a radiation pattern with significant sidelobe level reduction.

II. PROBLEM FORMULATION

Geometrical configuration is a key factor in the design process of an antenna array. For CCAAs, the elements are arranged in such a way that all antenna elements are placed in multiple concentric circular rings, which differ in radii and in number of elements. Fig. 1 shows the general configuration of CCAA with M concentric circular rings, where the m^{th} (m = 1, 2,…, M) ring has a radius r_m and the corresponding number of elements is N_m. If all the elements in all the rings are assumed to be isotopic sources, the radiation pattern of this array can be written in terms of its array factor only.

Referring to Fig.1, the array factor, $AF(\phi, I)$ for the CCAA in x-y plane may be written as (1) [7]:

$$AF(\phi, I) = \sum_{m=1}^{M}\sum_{i=1}^{N_m} I_{mi} \exp\left[j\left(kr_m \cos(\phi - \phi_{mi}) + \alpha_{mi}\right)\right] \tag{1}$$

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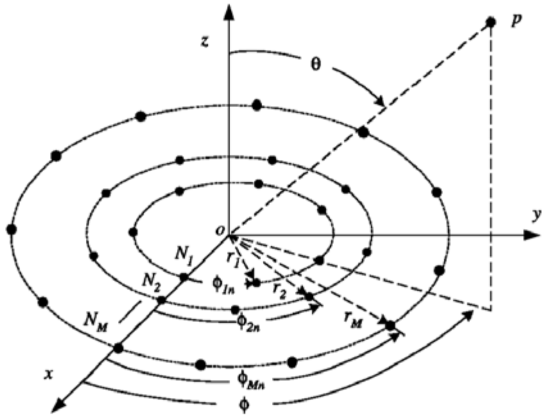

Fig. 1. Concentric circular antenna array (CCAA).

where I_{mi} denotes current excitation of the i^{th} element of the m^{th} ring. $k = 2\pi/\lambda$, λ being the signal wave-length. ϕ symbolize the azimuth angle from the positive x axis to the orthogonal projection of the observation point respectively. The angle ϕ_{mi} is the element to element angular separation measured from the positive x-axis. As the elements in each ring are assumed to be uniformly distributed, we have

$$\phi_{mi} = 2\pi\left(\frac{(i-1)}{N_m}\right); \qquad m = 1,\cdots,M; \quad i = 1,\cdots,N_m \tag{2}$$

α_{mi} is the phase difference between the individual elements in the array which is a function of angular separation ϕ_{mi} and ring radii r_m.

$$\alpha_{mi} = -Kr_m\cos(\phi_0 - \phi_{mi}); \qquad m = 1,\cdots,M; \quad i = 1,\cdots,N_m \tag{3}$$

where ϕ_0 is the value of ϕ where peak of the main lobe is obtained.

After defining the array factor, the next step in the design process is to formulate the objective function which is to be minimized. The objective function "Cost Function" (CF) may be written as (4):

$$CF = W_{F1} \times \frac{|AF(\phi_{msl1}, I_{mi}) + AF(\phi_{msl2}, I_{mi})|}{|AF(\phi_0, I_{mi})|} + W_{F2} \times |BWFN_{computed} - BWFN(I_{mi} = 1)| \tag{4}$$

where ϕ_0 is the angle where global maximum is attained in $\phi \in [-\pi, \pi]$. ϕ_{msl1} is the angle where the maximum sidelobe is attained in the lower band and ϕ_{msl2} is the angle where the maximum sidelobe is attained in the upper band. $BWFN$ is angular width between the first nulls on either side of the main beam. In (4) the two beamwidths, $BWFN_{computed}$ and $BWFN(I_{mi} = 1)$ are the first null beamwidths for the non-uniform excitation case and for the uniform excitation respectively. W_{F1} and W_{F2} are the weighting factors.

III. The Proposed Algorithms

A. Evolutionary Programming (EP)

In general, evolutionary computation programming is a very rich class of multi-agent stochastic search (MASS) algorithms based on the Neo-Darwinian paradigm of natural evolution, which can perform exhaustive searches in complex solution space [9]. These techniques start first with searching a population of feasible solutions generated stochastically; then, stochastic variations are incorporated to the parameters of the population in order to evolve the solution to a global optimum. Thus, these methods provide a rigorous stochastic search in the entire domain taking into account maximum possible interactions among them.

B. Basic Evolutionary Programming (BEP)

The efficiency of any optimization algorithm for finding a global optimum for a given function depends on how effectively the balance has been maintained between two contradictory requirements, exploitation of already discovered regions, and exploration of new and unknown regions in the search space. An evolutionary programming method, which models evolution at the level of computing species for the same resources, uses mutation as the sole operator for the advancement of generation, and the amount of exploitation and exploration is decided only through the mutation operator. Usually, in the BEP [9], the mutation operator produces one off-spring from each parent by adding a Gaussian random variable with zero mean and a variance proportional to the individual fitness score. The value of standard deviation, which is the square root of the variance, decides the characteristics of off-spring produced with respect to its parent. A standard deviation close to zero will produce off-spring that has more probability of resembling its parent, and a much less probability of being largely or altogether different from it. As the value of the standard deviation departs from zero, the probability of resemblance of off-spring with its parent decreases, and the probability of producing altogether different off-spring increases. With this feature, the standard deviation essentially maintains the trade-off between exploration and exploitation in a population during the search.

The main steps of the algorithm are as follows:

Step 1: Initialization:

Initialize a population pool. Let $I_i = [I_{1,i}\ I_{2,i}\ I_{3,i}\ \ldots.\ I_{j,i} \ldots.\ I_{n,i}]$, where j=1 to n; n is the number of current excitation weights.

Step 2: Creation of BEP based off springs

An i^{th} off-spring vector I'_i is created (mutation process) from each parent vector I_i by adding to each component j of I_i a Gaussian random variable N (0, $\sigma_j(i)$) with zero mean and a standard deviation $\sigma_j(i)$ proportional to scaled CF values of the parent trial solution as given by the following equations:-

$$I'_i = [I'_{1,i}\ I'_{2,i}\ I'_{3,i}\ \ldots\ldots\ldots\ I'_{j,i} \ldots\ldots\ I'_{n,i}] \tag{5}$$

where $I'_{j,i} = I_{j,i} + N(0, \sigma_j(i))$

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$$\sigma_j(i) = \alpha \times \frac{CF(i)}{\min\_CF} \times \left(I_{j,i\max} - I_{j,i\min}\right)$$

where "$\min\_CF$" is minimum CF value in the population n_p and α is the strategy parameter experimentally chosen within prefixed limits and remains fixed for the maximum number of BEP run cycles. Population of off-spring vectors is also n_p. So, a total population pool of $2 \times n_p$ vectors is formed and all will undergo mutual competition and selection through random evaluation of a quantity called 'score' (Step 3).

Step 3: Evaluation, Competition and Selection

The n_p parent trial vectors I_i and n_p corresponding off-spring vectors I'_i contend for surviving with each other within the competition pool of size $2 \times n_p$. The score for ith trial vector in $2 \times n_p$ pool is evaluated by

$$w_i = \sum_{j=1}^{n_p} w_j \tag{6}$$

w_j =1, if ($CF(i) < CF(t(j))$) else w_j = 0, where j varies from 1 to n_p, i varies from 1 to $2 \times n_p$ and t(j)= $[2 \times n_p \times u + 1]$, u is random number ranging over [0, 1] and [y] denotes the greatest integer less than or equal to y.

After competing, the $2 \times n_p$ trial solutions including the parents and the off-springs are ranked in descending order of the score obtained in (6). The first n_p survive and are transcribed along with their objective functions CF into the survivor set as the basis of the next generation. A maximum number of BEP run cycles 'N_m' is given. The search process is stopped as the final count of run cycles reaches 'N_m'. Then, the optimal cycle is determined for which the CF is the grand lowest and the solution is optimal.

IV. Hybrid Evolutionary Programming (HEP)

Steps 1 and 2 are the same. Step 3 involves creation of another set of off-springs using FBM operator (FBM based off-springs), which will be actually competing with BEP based off-springs instead of parent solutions as in BEP algorithm.

In the HEP [9] algorithm, the primary factor is the introduction of a new mutation mechanism, which is called a fitness-blind mutation (FBM) operator. The FBM operator uses a standard deviation ($\sigma_{FBMj,i}$) for the Gaussian distribution, which is made proportional to the absolute value of its genotype distance ($Diff_{j,i}$) from the fittest parent in that generation by another strategy multiplying factor β as given below:

$$Diff_{j,i} = \text{absolute value of } (I_{jopt} - I_{j,i}) \tag{7}$$

where j is the number of parameters to be optimized and I_{jopt} is the parameter corresponding to minimum CF among n_p trial vectors for the current BEP run cycle and $I_{j,i}$ are the parameters of ith trial vector of the same cycle.

$$\sigma_{FBM\,j,i} = \beta \times Diff_{j,i} \tag{8}$$

The '+'or '-'sign of ($I_{jopt} - I_{j,i}$) is also computed as $Dir_{j,i}$. The signs are not necessarily the same for different parameters denoted by j.

An FBM operated off-spring vector $I_{j,i}''$ is created from each parent $I_{j,i}$ by mutation, which is essentially algebraically adding (depending on the sign as computed) to each component (j) of I_i a Gaussian random variable with zero mean and the standard deviation $\sigma_{FBM\,j,i}$.

$$I_{j,i}'' = I_{j,i} + N(0, \sigma_{FBM\,j,i}) \times Dir_{j,i} \tag{9}$$

Then, evaluation, competition and selection are similarly performed among the BEP generated off-springs and FBM generated off-springs (instead of parents as in BEP), the total population pool being the same as $2 \times n_p$. Effective strategy parameters, α, β are fixed by experimentation.

V. Numerical Results

This section gives the experimental results for various CCAA designs obtained by HEP technique. For this optimization technique ten three-ring (M=3) CCAA designs for each of two cases as a) without central element feeding and b) with central element feeding in three-ring CCAA are assumed. Each CCAA maintains a fixed spacing between the elements in each ring (inter-element spacing being 0.55λ, 0.61λ and 0.75λ for first ring, second ring and third ring respectively). These spacings are the means of the values determined for the ten designs for non-uniform spacing and non-uniform excitations in each ring using 25 trial generalized optimization runs for each design. For all sets of experiments, the number of elements for the inner most ring is N_1, for the outermost ring is N_3, whereas the middle ring consists of N_2 number of elements. For all the cases, $\phi_0 = 0^0$ is considered so that the centre of the main lobe in radiation patterns of CCAA starts from the origin. After several experimentations, the best proven parameters are: i) W_{F1} and W_{F2} are fixed as 18 and 1 respectively. ii) Initial population = 120, iii) Maximum number of iteration cycles = 400, and iv) $\alpha = 0.4$, $\beta = 0.6$.

Sets of three-ring CCAA (N_1, N_2, N_3) designs considered for both without and with central element feeding are (2,4,6), (3,5,7), (4,6,8), (5,7,9), (6,8,10), (7,9,11), (8,10,12), (9,11,13), (10,12,13), (11,13,15). HEP generates a set of normalized, optimized non-uniform current excitation weights for each CCAA design. Some of the optimal results as determined by HEP are shown in Tables II-III. Table I depicts SLL values and $BWFN$ values for all corresponding uniformly excited (I_{mi}=1) CCAA designs.

A. Analysis of Radiation Pattern of Optimal CCAA

Figs. 2-3 depict the substantial reductions in SLL with non-uniform optimal current excitations as compared to the case of uniform non-optimal current excitations. All CCAA design sets having central element feeding (Case (b)) yield much more reductions in SLL as compared to the same not having central element feeding (Case (a)). As shown in Tables II-III, SLL reduces to -32.4 dB for Case (a) and **-40.22 dB (grand lowest SLL) for Case (b),** with the CCAA having N_1=4, N_2=6, N_3=8 elements (Set No. III). $BWFN$ become narrower for non-uniform optimal

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current excitations as compared to the case of uniform non-optimal current excitations for all the design sets in both the test cases. For the same optimal CCAA set, the *BWFN* values are 77.1^0 for Case (a), and 93.2^0 for Case (b) against 90.3^0 (Case (a)), 95.4^0 (Case (b)) respectively for the corresponding uniformly excited CCAA having the same number of elements. So, this technique yields maximum reductions of *BWFN* also for this optimal CCAA.

B. *Convergence profile for HEP*

The minimum *CF* values are recorded against the number of iteration cycles to get the convergence profile. Fig. 4 portrays the convergence profile of minimum *CF* for N_1=4, N_2=6, N_3=8 with central element feeding. The programming is written in MATLAB 7.5 version on core (TM) 2 duo processor, 3.00 GHz with 2 GB RAM.

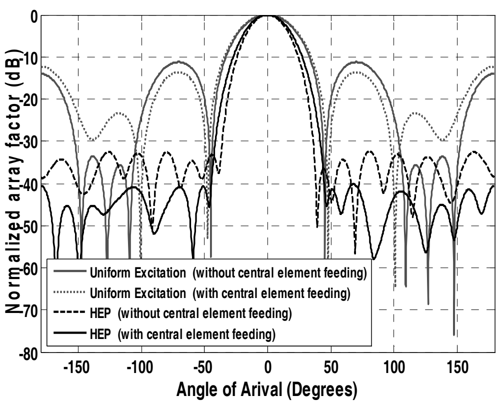

Fig. 2. Radiation pattern for a uniformly excited CCAA and corresponding HEP based non-uniformly excited CCAA (N_1=4, N_2=6, N_3=8).

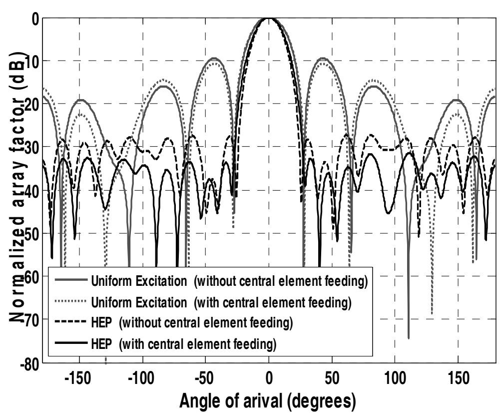

Fig. 3. Radiation pattern for a uniformly excited CCAA and corresponding HEP based non-uniformly excited CCAA (N_1=8, N_2=10, N_3=12).

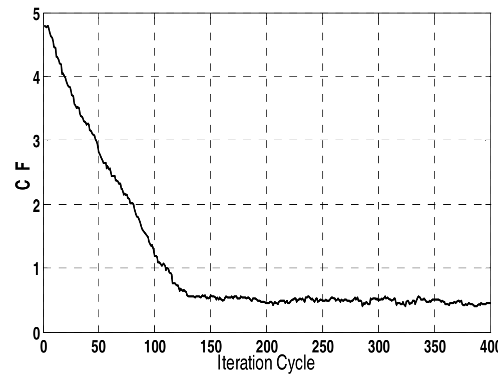

Fig. 4. Convergence profile for HEP in case of non-uniformly excited CCAA (N_1=4, N_2=6, N_3=8 with central element feeding)

Table I

SLL And BWFN For Uniformly Excited (I_{mi} =1) CCAA Sets

Set No.	No. of elements in each rings (N_1,N_2,N_3)	Without central element (Case (a))		With central element (Case (b))	
		SLL (dB)	BWFN (deg)	SLL (dB)	BWFN (deg)
I	2, 4, 6	-12.56	128.4	-17.0	140.0
II	3, 5, 7	-13.8	107.2	-15.0	116
III	4, 6, 8	-11.23	90.3	-12.32	95.4
IV	5, 7, 9	-11.2	78.2	-13.24	81.6
V	6, 8, 10	-10.34	68.4	-12.0	71.1
VI	7, 9, 11	-10.0	61.0	-11.32	63.0
VII	8, 10, 12	-9.6	54.8	-10.76	56.4
VIII	9, 11, 13	-9.28	50.0	-10.34	51.3
IX	10, 12, 14	-9.06	46.0	-10.0	47.0
X	11, 13, 15	-8.90	42.0	-9.8	43.2

Table II

Current Excitation Weights, SLL and BWFN For Non-Uniformly Excited CCAA Design Sets (Case (a)) Using HEP

Set No.	Current excitation weights for the array elements (I_{11} , I_{12} ,…., I_{mi})	*CF*	SLL (dB)	BW FN (deg)
I	0.6436 0.6539 0.6168 0.0902 0.6119 0.1027 0.2256 0.2286 0.0816 0.2298 0.2301 0.0980	1.83	-29.0	126.7
III	0.0192 0.4230 0.0233 0.4009 0.2530 0.2507 0.6606 0.2746 0.2473 0.6098 0.2956 0.4095 0.3052 0.1664 0.3213 0.4082 0.3124 0.1514	0.88	-32.4	78.3
V	0.2539 0.1643 0.2535 0.1615 0.1837 0.4012 0.4195 0 0.3601 0.5418 0.3092 0 0.4333 0.5739 0.1976 0.4085 0.4174 0.2424 0.3586 0.2639 0.3406 0.4188 0.2275 0.2457	1.78	-25.96	59.3
VII	0.3356 0.1697 0.2629 0.2719 0.3039 0.1342 0.3562 0.4342 0.3223 0.0657 0.0444 0.1575 0.1418 0.1738 0.0445 0.0721 0.3286 0.2282 0.1509 0.1533 0.3812 0.0685 0.1776 0.1759 0.1427 0.1064 0.3736 0.1897 0.1265 0.1526	1.53	-27.30	51.3

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Table III
Current Excitation Weights, SLL and BWFN For Non-Uniformly Excited CCAA Design Sets (Case (b)) Using HEP

Set No.	Current excitation weights for the array elements ($I_{11}, I_{12}, \ldots, I_{mi}$)	*CF*	SLL (dB)	BW FN (deg)
I	0.0008 0.3528 0.3569 0.3323 0.0724 0.3329 0.0758 0.1377 0.1405 0.0411 0.1319 0.1291 0.0495	1.81	-29.54	128.8
III	0.3439 0.5294 0.4568 0.5139 0.4515 0.3531 0.3480 0.0396 0.3428 0.3411 0.0300 0.1236 0.2012 0.1318 0.0177 0.1428 0.2002 0.1390 0.0189	0.39	**-40.22**	93.2
V	0.1939 0.2532 0.2000 0.1735 0.2011 0.2319 0.3748 0.3846 0.0016 0.2338 0.4292 0.2103 0.0127 0.3924 0.4618 0.1867 0.4303 0.3675 0.1651 0.2062 0.1539 0.3414 0.4194 0.1625 0.1976	1.39	-28.8	58.8
VII	0.4078 0.1970 0.4477 0.1998 0.5073 0.2234 0.4604 0.1883 0.3547 0.2062 0.0260 0.0298 0.3548 0.1008 0.3343 0 0.0108 0.2729 0.0710 0.1091 0.0752 0.2589 0.1412 0.1478 0.0961 0.1405 0.1487 0.2681 0.1353 0.0947 0.0398	1.02	-31.52	58.1

VI. Conclusion

In this paper, the optimal design of a non-uniformly excited CCAAs with uniform inter-element spacing and with / without central element feeding has been described using the hybrid evolutionary optimization technique, HEP. Experimental results reveal that the design of non-uniformly excited CCAA offer a considerable SLL reduction along with the reduction of *BWFN* as well as compared to the case of corresponding uniformly excited CCAA. The main contribution of the paper is twofold: (i) All CCAA designs having central element feeding yield much more reductions in SLL as compared to the same not having central element feeding, (ii) The CCAA design having N_1=4, N_2=6, N_3=8 elements along with central element feeding gives the grand maximum SLL reduction **(-40.22 dB)** as compared to all other designs, which one is thus the grand optimal design among all the three-ring designs. Thus, the proposed HEP technique proves to be a promising evolutionary optimization technique for the global optimization of antenna array problem.

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